ON OUT-OF-EQUILIBRIUM PHENOMENA IN PSEUDOGAP PHASE OF COMPLEX SYK+U MODEL

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In a nutshell

In [1], [2] it was demonstrated that in complex SYK+U model in the limit of small Hubbard interaction, $U \ll J$ (here U is the Hubbard interaction constant and J is the SYK model interaction constant) the pseudogap phase appears. This phase corresponds to a non-trivial saddle-point in mean-field treatment, where the phases of gaps $\Delta_i = |\Delta| e^{i\theta_i}$ are not fixed by saddle point equation. These phases are soft degrees of freedom and their fluctuations can destroy off-diagonal long-range order (ODLRO) even in $N \to \infty$ limit. The pseudogap phase corresponds to the non-synchronized Cooper pairs. We shall show that dynamics of the phase mode in SYK+U model is identical to the quantum Hamiltonian mean field (HMF) model. Using this fact, we shall demonstrate that well-known out-of-equilibrium properties of classical HMF model, like existence of long-living quasi-stationary states (QSS), persist on quantum level and can cause residual superconductivity in pseudogap phase of SYK+U model. The detailed discussion can be found in [3]

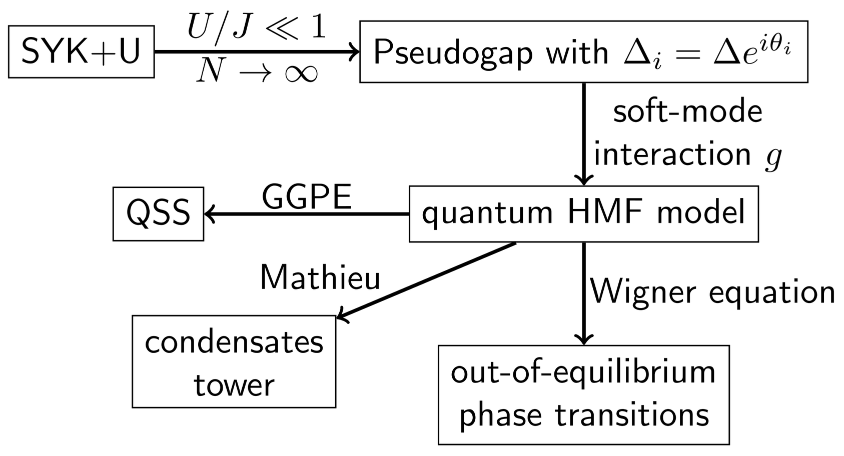

Figure 1: In a nutshell

Set-up and model Hamiltonian

The model involves fermions with SYK interaction,

$$H_{\text{SYK}} = = \frac{1}{2} \sum_{ijkl,\sigma,\sigma'}^{N} J_{ijkl} \left[c_{i\sigma}^\dagger c_{j\sigma'}^\dagger c_{k\sigma'} c_{l\sigma} + c_{l\sigma}^\dagger c_{k\sigma'}^\dagger c_{j\sigma'} c_{i\sigma} \right], \tag{1}$$

where J_{ijkl} are assumed to be real independent random variables, drawn from the Gaussian distribution with the mean $\langle J_{ijkl} \rangle = 0$ and variance $\langle J_{ijkl}^2 \rangle = J^2/(4N)^3$. It is supplemented with the attractive Hubbard interaction

$$H_{\text{U}} = -U \sum_{i}^{N} c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger c_{i\downarrow} c_{i\uparrow} - \mu \sum_{i,\sigma}^{N} c_{i\sigma}^\dagger c_{i\sigma}. \tag{2}$$

The dimensionless parameter U/J characterizes attraction strength. Introducing two-point fields,

$$G(\tau,\tau') = -\frac{1}{N} \sum_{i=1}^{N} c_{i\sigma}(\tau) c_{i\sigma}^\dagger(\tau'), \quad F(\tau,\tau') = -\frac{1}{N} \sum_{i=1}^{N} c_{i\downarrow}(\tau) c_{i\uparrow}(\tau') \tag{3}$$

and performing decoupling in Cooper channel, the action for SYK+U model becomes

$$S = \sum_{i=1}^{N} \int d\tau \left[\frac{|\Delta_i|^2}{U} - \frac{1}{2} \operatorname{Tr} \ln \left(\partial_\tau + \mu + \hat{\Sigma} + \hat{\Delta}_i \right) \right] - \\ - N \int \int d\tau \, d\tau' \left[\hat{\Sigma}(\tau,\tau') \hat{G}(\tau',\tau) + \frac{J^2}{64} \left(F(\tau,\tau')^2 F(\tau,\tau')^2 + G(\tau,\tau')^4 \right) \right] \tag{4}$$

where $\hat{\Sigma}$ and $\hat{G}$ are Nambu-like matrix fields and $\hat{\Delta}_i = \Delta_i \sigma_+ + \overline{\Delta}_i \sigma_-$. Averaging over disorder followed by saddle-point ansatz in large-N limit raises after variation the following mean-field equations for Green functions,

$$G(\omega_n) = \frac{-i\omega_n + \Sigma(\omega_n)}{[\omega_n + i\Sigma(\omega_n)]^2 + \Delta^2}, \quad F(\omega_n) = -\frac{\Delta}{[\omega_n + i\Sigma(\omega_n)]^2 + \Delta^2}, \tag{5}$$

which are valid for exponentially small Δ. Solution of the analog of gap equation gives the following value of mean-field gap Δ,

$$\Delta \propto \begin{cases} J \exp(-J/U), & U \ll J, \\ U/2, & U \gg J. \end{cases} \tag{6}$$

However, the saddle-point equations do not fix the gap phase, i.e. we have $\Delta_i = \Delta \exp(i\theta_i)$, where Δ can be determined via equations above. Phases of Δ_i correspond to soft degrees of freedom and they can destroy ODLRO even in $N \to \infty$ limit. At this stage the pseudogap phase appears: we have non-zero value of gap Δ, but the phases of Cooper pairs are not synchronized, which prevents the superconducting state. The pseudogap effective action becomes

$$S_{\text{eff}} = \int d\tau \left[\frac{m}{2} \sum_{i=1}^{N} \dot{\theta}_i^2 - \frac{g}{N} \sum_{i<j} \cos(\theta_j - \theta_i) \right], \tag{7}$$

where the phase interaction constant g is computed from the off-diagonal Cooper susceptibility and $m \propto 1/J$ is related to the susceptibility of ground state energy E_{GS} to a local chemical potential μ. Computation of ladder diagrams gives $g \propto J \exp(-\pi J/U) > 0$.

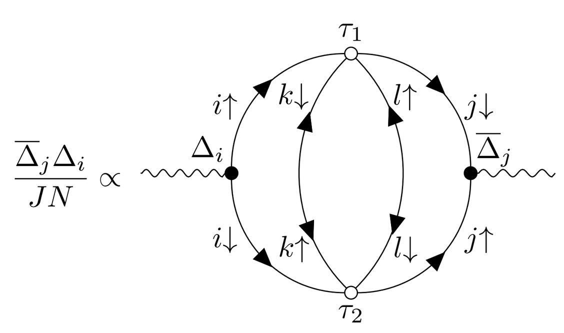

Figure 2: First contribution to off-diagonal Cooper susceptibility (adapted from [1])

Pseudogap Phase Action

The effective action after Wick rotation raises the famous Hamiltonian mean-field model,

$$H_{\text{HMF}} = -\frac{1}{2m} \sum_{i=1}^{N} \frac{\partial_i^2}{\partial \theta_i^2} - \frac{g}{N} \sum_{i<j} \cos(\theta_j - \theta_i). \tag{8}$$

The classical HMF model exhibits the continuous phase transition between non-synchronized and synchronized states with order parameter, $M = \sum_{j=1}^{N} e^{i\theta_j}/N$. The synchronized state has $|M| \neq 0$, whereas the non-synchronized state has $|M| = 0$. For the quantum HMF model at low temperatures, we can use generalized Gross-Pitaevskii equation (GGPE) to study properties of the model,

$$i\chi \frac{\partial \Psi}{\partial \tau} = -\frac{\chi^2}{2} \frac{\partial^2 \Psi}{\partial \theta^2} - \Phi(\theta,\tau)\Psi, \Phi(\theta,\tau) = \int_{-\pi}^{+\pi} d\theta' \cos(\theta - \theta') \, |\Psi(\theta,\tau)|^2, \tag{9}$$

where we have introduced rescaling Planck constant $\chi = 1/\sqrt{mg}$ and also rescaled time, $\tau = t/\sqrt{mg}$. The GGPE (9) can be rewritten as quantum Euler equations with representing $\Psi = \sqrt{\rho} e^{iS/\chi}$ and $v = \partial S/\partial\theta$,

$$\frac{\partial \rho}{\partial \tau} + \frac{\partial}{\partial \theta}(\rho v) = 0, \quad \frac{\partial v}{\partial \tau} + v \frac{\partial v}{\partial \theta} - \frac{\partial \Phi}{\partial \theta} = -\frac{\partial Q}{\partial \theta}, \quad Q \equiv \frac{\chi^2}{2\sqrt{\rho}} \frac{\partial^2 \sqrt{\rho}}{\partial \theta^2}. \tag{10}$$

Linear stability analysis of small perturbation $\delta\rho = \delta\rho(\theta,\tau)$ near incoherent state $\rho_0 = (2\pi)^{-1}$ tells us that homogeneous solution ρ_0 is stable if $\chi > \chi_c = \sqrt{2}$. As should be, this result coincides with $g_c = (2m)^{-1}$ (in terms of g, the homogeneous solution ρ_0 is stable for $g < g_c$).

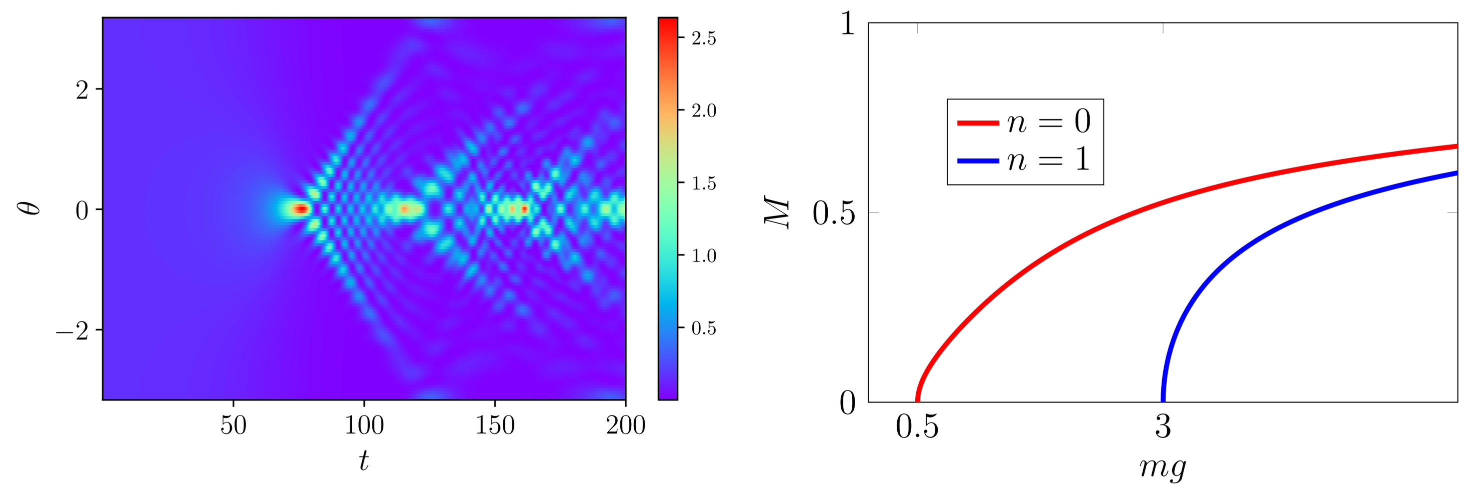

Figure 3: Left: QSS in SYK+U model with $\chi = 0.1$; Right: equilibrium phase diagram

The stationary spatially inhomogeneous density $\rho = \rho(\theta)$ for synchronized state can be obtained via self-consistent solving of Mathieu equation. This procedure shows that there is a tower of phase condensates, which characterized by Mathieu functions with different number of nodes n. For the classical HMF model, it was show that in $N \to \infty$ the quasi-stationary states with large lifetime appear and prevents thermalization. In case of quantum HMF model, QSS still alive for small enough χ. Existing of such QSS on quantum level means that fingerprints of superconductivity persist in pseudogap phase, too. In terms of Cooper pair phases, these QSS correspond to partially synchronized states.

Summarizing the mapping of out-of-equilibrium phenomena in classical HMF model onto the pseudogap phase in SYK+U model, we propose:

Tower of condensates: stationary solutions of GGPE can be found by considering the corresponding Mathieu equation,

$$-\frac{\chi^2}{2} \frac{\partial^2 \Psi}{\partial \theta^2} + (-\mu - M \cos\theta) \Psi = 0, \quad M = \int_{-\pi}^{+\pi} d\phi \, |\Psi(\phi)|^2 \cos(\phi - \theta), \tag{11}$$

assuming that M is a given constant. The higher condensates are represented by Mathieu functions ce_n for even n and se_{n+1} for odd n.

Quantum QSS: for quite small values of χ, in the pseudogap phase of SYK+U model, long-living QSS exist and they can be captured by the analysis of phases θ_i quench. We prepare the initial state with given $\theta_i(0)$ and $\dot{\theta}_i(0)$ and see how this quench evolves in time, producing quantum QSS

Out-of-equilibrium phase transitions: in the pseudogap, there is a out-of-equilibrium phase transition between non-synchronized and partially synchronized state. This transition can be captured by the Wigner function analysis and it happens with respect to initial conditions for the Wigner function, which encode initial order parameter value M_0 and the energy per particle E. Varying M_0 and E, one can find the critical line, which separates two different QSS with $|M_{\text{QSS}}| = 0$ and $|M_{\text{QSS}}| \neq 0$.

References

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